Neutrinos from Supernovas and Supernova Remnants

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Abstract. Supernovae (SN) and supernova remnants (SNR) have key roles in galaxies, but their physical descriptions is still incomplete. Thus, it is of interest to study neutrino radiation to understand SN and SNR better. We will discuss: (1) The ∼10 MeV thermal neutrinos that arise from core collapse SN, that were observed for SN1987A, and can be seen with several existing or planned experiments. (2) The 10-100 TeV neutrinos expected from galactic SNR (in particular from RX J1713.7-3946) targets of future underwater neutrino telescopes.

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1. INTRODUCTION

In the economy of a galaxy, the stars with mass $\geq 6-10\ M_\odot$ are known to play a prominent role. The last instants of their brief life (on ten million year scale) is a crucial moment. In fact, the gravitational collapse of the 'iron' core leads to supernovae of type II, Ib, Ic and eventually to pulsars, neutron stars and stellar black holes. The gas subsequently expelled by all supernovas including Ia (that lead to the observed supernova remnants) is suspected to be the place where cosmic rays are accelerated. A complete theoretical understanding of these systems (and in particular of the supernova) is not yet available. However, it is known that neutrinos of ~ 10 MeV are emitted during and just after the gravitational collapse, and that neutrinos above 1 TeV can be produced in the supernova remnant at least in special circumstances. Therefore, neutrinos can be used as diagnostic tools of what happens in these interesting and complex systems. With these considerations in mind, we will discuss here briefly neutrinos from supernova and from supernova remnants, aiming to outline the main 'what', 'when', 'where', and 'how' questions.

Outline and references. In the rest of this section, we discuss a number of general facts pertaining to supernovas and supernova remnants; the references that we use are [1, 2]. In the second section we concentrate on supernova neutrinos; we use [1, 2], and also [3]. In the third section we describe the calculations of [4]. In these 4 papers, one can to find more details and a complete list of references. In the appendix, we show in details a simple way to calculate neutrinos from cosmic rays, adopted also in [4].

Generalities. Before passing to neutrinos, we would like to discuss a few general questions.

Guessing 'where' Let us recall that the only SN within the capability of existing neutrino detectors are those from our Galaxy (or some irregular galaxy around us). The best guess we can propose for next galactic core collapse SN is

$$\langle L\rangle = 10 \pm 4.5 \text{ kpc} \tag{1}$$

and it is motivated as follows:
1) We are $R = 8.5$ kpc from the galactic center.
2) The distribution of the matter that can go in supernovae is: $\rho \sim re^{-r/r_0}$ with r=distance from the center and $r_0 = 3$ kpc, possibly summing a $\delta(r)$ to describe the 'bar'
3) We calculate the distribution in function of L=distance from us, integrate over the galactic azimuth θ and get the result above.

Guessing 'when' The rate of core collapse SN in the Milky Way is expected to be

$$R_{SN} = 1/(\text{30-70 years}) \tag{2}$$

The most reliable method is: count SN in other galaxies, and correlate with galactic type.[1] A similar rate expected for SN Ia. Possibly, we missed several galactic SN due to dust.[2] For the future, we will have better coverage of galactic SN with neutrinos, IR and perhaps with gravitational waves. From the absence of neutrino bursts, one derives

$$R_{SN} > 1/(21 \text{ years}) \text{ at } 1\sigma \tag{3}$$

We assume Poisson statistics (namely, $\exp(-TR_{SN}) = 1 - C.L$) with $T = 24$ years. To obtain the last number,

[1] The Padova-Asiago database [5] includes several thousand SN. The Milky Way could be Sb or Sb/c, which means a factor 2 uncertainty.
[2] Indeed, with about 10 supernovae seen in last 2000 years, in order to have a SN each 25 years we need to admit we see just one SN each 8, that is about the right figure on accounting the presence of the dust and the possibility that a supernova explodes on the day sky [6].

we note that till 1986 only Baksan [7] worked with 90 % DAQ livetime, then we assume 100 % coverage.

We conclude by listing the number of SN precursors and descendants. We estimated these populations by assuming $R_{SN}^{tot} = 1/(25 \text{ years})$.

Object	Lifetime	Number
Pre-SN with ν	20 million y	400.000
Pulsars	2 million y	40.000
SNR	100.000 y	2×2000
young SNR	2000 y	2×40

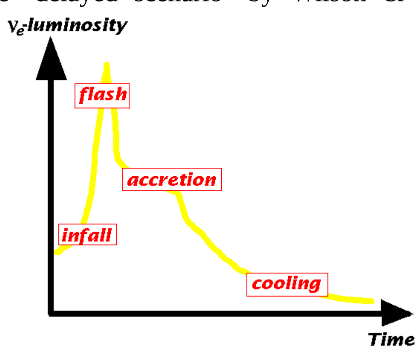
By 'Pre-SN with ν' we mean core collapse SN. Recall that core collapse SN produce neutron stars (NS) or stellar black holes, and that pulsars are 'active' NS. Type Ia SN make white dwarfs, instead, but are also supposed to produce SNR (which explains the factor 2 above).

2. νS FROM SUPERNOVAE

We begin recalling the astrophysics of core collapse. The giant stars–**pre-SN**–burn in sequence H, He, C and Si, Ne, Mg, Na *etc*, and form an "onion structure", with an inert 'iron' core. Violent stellar winds occasionally modify external **mantle** in latest stages; apparently, this happened for SN1987A that was a $\sim 20\ M_\odot$ *blue* giant.What happens in the **core**? The gravitational pressure is balanced by e^- degeneracy pressure, and the core grows. As demonstrated by Chandrasekhar, the pressure of free electrons is unable to supply an equilibrium configuration when e^- become relativistic. Now, the iron core mass is $\sim 1.4\ M_\odot$, the **collapse** begins and the sequence of the events becomes uncertain (more on the reference picture, the so-called "delayed scenario"). What about the energetic of the collapse? The total energy of the collapse is very large, and 99 % of this energy is carried away by neutrinos. With $M_{ns}/M_\odot = 1-2$, $R_{ns} = 15\ \text{km}(M_\odot/M_{ns})^{1/3}$, we estimate

$$\mathscr{E} \simeq \frac{3 G_N M_{ns}^2}{7 R_{ns}} = (1-5)\times 10^{53}\ \text{erg}$$

The delayed scenario. This is a pictorial summary of the 'delayed scenario' by Wilson & Bethe [8]:



The energy radiated in any neutrino species $e, \bar{e}, \mu, \bar{\mu}, \tau, \bar{\tau}$ is expected to be the same within a factor of two [9]:

$$\mathscr{E}_e \sim \mathscr{E}_{\bar{e}} \sim \mathscr{E}_x$$

where x denotes any among μ, $\bar{\mu}$, τ, $\bar{\tau}$. Indeed, in this picture non-electronic neutrinos and antineutrinos are produced in a similar amount.

Note that neutrinos are mostly emitted in cooling (80-90 %) and accretion (10-20 %) phases. The latter one is the phase crucial to understand the nature of the explosion, whereas the first phase is almost thermal. In this sense, the ignorance of the detailed mechanism of the explosion does not preclude a description of the bulk of neutrino radiation (to put it with a slogan, ignorance is self-consistent).

Now we come to a prescription for the fluence:

$$F_i(E) = \frac{\mathscr{E}_i}{4\pi L^2} \frac{N}{\langle E_i \rangle^2} z^\alpha e^{-(\alpha+1)z}, \qquad z = \frac{E}{\langle E_i \rangle}$$

where $\langle E_i \rangle$ is the average energy of the neutrino species $i = e, \bar{e}, x$; N ensures that the total energy carried is $\mathscr{E}_i$. (If one needs to describe time dependent situations, $\mathscr{E}_i \to L_i(t) \equiv d\mathscr{E}_i/dt$, $\langle E_i \rangle \to \langle E_i(t) \rangle$, $\alpha \to \alpha(t)$.) The expectations for time integrated quantities are:

$$\begin{array}{ll} \langle E_{\bar{e}} \rangle = 12-18\ \text{MeV}, & \langle E_x \rangle / \langle E_{\bar{e}} \rangle = 1-1.2 \\ \mathscr{E}_{\bar{e}} = (2-10)\times 10^{52}\ \text{erg} & \mathscr{E}_x / \mathscr{E}_{\bar{e}} = 1/2-2 \end{array}$$

One guesses $\mathscr{E}_e = \mathscr{E}_{\bar{e}}$ (not so important); the ν_e average energy can be estimated from the emitted lepton number.

Oscillations of supernova neutrinos. Oscillations of SN neutrinos are pretty simple to describe. To account for oscillations we need to assign just 2 functions, P_{ee} and $P_{\bar{e}\bar{e}}$:

$$\begin{array}{rl} \bullet F_e & = F_e^0\, P_{ee} + F_\mu^0\, P_{\mu e} + F_\tau^0\, P_{\tau e} \\ & = F_e^0\, P_{ee} + F_x^0\, (1-P_{ee}) \\ \bullet F_e + F_\mu + F_\tau & = F_e^0 + F_\mu^0 + F_\tau^0 \end{array}$$

and similarly for antineutrinos (we will consider only oscillations of massive νs). The relevant densities to calculate P_{ee} and $P_{\bar{e}\bar{e}}$ are $\rho_{sol} \sim 10$ gr/cc (He) and $\rho_{atm} \sim 10^3$ gr/cc (C+O).

There is a great interest on the unknown parameter U_{e3}, and supernovas offer a chance to learn more on that. However, we should keep in mind the uncertainties mentioned above. Let us try to discuss this point further. We begin with the formula $F_e = F_x^0 - P_{ee}(F_x^0 - F_e^0)$. If we have normal mass hierarchy:

$$P_{ee} = \left\{ \begin{array}{ll} \sin^2\theta_{13} \sim 0 & \theta_{13}\ \text{'large'},\ > 1^\circ \\ \sin^2\theta_{12} \sim 0.3 & \theta_{13}\ \text{'small'},\ < 0.1^\circ \end{array} \right.$$

Now we can ask a precise question on U_{e3}:

Can we distinguish the two cases?

TABLE 1. Depending on the phase of neutrino emission, it is possible to learn more on U_{e3} studying of SN neutrinos. In the table, goods and bads of each specific phase are mentioned. (LSD refers to the observations of the Mont Blanc detector, that according to Imshennik and Ryazhskaya [10] could be interpreted assuming an intense neutronization phase before the main ν burst.)

Emission	Good	Bad	Remarks
cooling	strong ν radiation	uncertainties, small effect	$F_x^0 \sim F_e^0$
accretion	strong ν radiation	uncertainties!	$F_x^0 \sim F_e^0/2$
neutronization	clean signal	weak ν radiation	$F_x^0 \sim 0$
neutroniz.++ with rotation	clean and strong sign.	uncertain!!	$F_x^0 \sim 0$ *(LSD?)*

Table 1 provides a check-list. Note that if the mantle is stripped off till densities $\rho > 10$ gr/cc (e.g., with SN Ic) we have vacuum oscillations, $0.3 \to 0.6$ (Selvi). This is rare, but not impossible.

SN1987A. The first detection of SN neutrino is of epochal importance. These observations fit into the 'standard' picture for neutrino emission (see next figure), but there are some puzzling aspects:
1. IMB [11] and Kamiokande-II [12] find forward peaked distributions; e.g., $\langle \cos\theta \rangle$ are $\sim 2\,\sigma$ above expectations.
2. $\langle E_{vis}^{\rm KII} \rangle \sim 15$ MeV and $\langle E_{vis}^{\rm IMB} \rangle \sim 30$ MeV (± 2.5 MeV) are quite different even correcting for efficiencies.
3. The time sequence of events looks different; when combined not so bad (but abs. time is unknown).
4. The 5 LSD events, occurred 4.5 hours before the main signal, cannot be accounted for.
In figure 1, will stress the consistency of the 'standard' interpretation, but the space for non-standard ones is not small (not only due to limited statistics). A reasonable agreement with expectations is obtained if:

$$\langle E_{\bar{e}} \rangle \equiv E_0 = 12 - 16\ \mathrm{MeV}$$
$$\mathscr{E} = (2-3) \times 10^{53}\ \mathrm{erg}$$
zero or a few $\nu_e\, e \to \nu_e\, e$ events in KII

3. νS FROM SNR

The leftover gas is the SNR. The kinetic energy is a few times 10^{51} erg, which means that the gas is in free expansion with velocities of about ~ 4000 km/sec. There are various phases of the SNR, according to their age, and various shapes (shell, plerionic, or mixed).

An argument by Ginzburg and Syrovatskii suggests SNR as main source of galactic cosmic rays (CR):
1) The Milky Way irradiates CR. Take $V_{CR} = \pi R^2 H$ with $R \sim 15$ kpc, $H \sim 5$ kpc as the volume of confinement. Take $\tau_{CR} = 5 \times 10^7$ years as CR lifetime in the Galaxy. We get the CR luminosity:

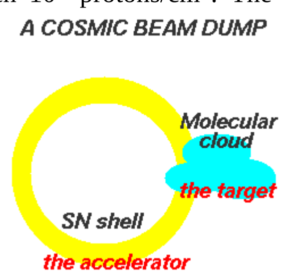
$$\mathscr{L}_{CR} = \frac{V_{CR} \cdot \rho_{CR}}{\tau_{CR}} = 0.9 \times 10^{41} \mathrm{erg/s}$$

2) We have a new SN each $\tau_{SN} \sim 25$ year, with about $\mathscr{E} \sim 10^{51}$ erg in kinetic energy, that is

$$\mathscr{L}_{SN} = \frac{\mathscr{E}}{\tau_{SN}} = 1.2 \times 10^{42} \mathrm{erg/s}$$

Comparing the two formulae, we see that if a SNR is able to convert a fraction $f_{CR} \sim 5-10$ % of the injected energy into CR, we are home (the numbers quoted above shouldn't be taken too seriously, but this 40-years-old argument maintains its appeal).

We proceed describing the system. In 2000 years, the SNR proceeds by ~ 10 pc. The density is ~ 0.2 protons/cm^3, too faint to permit a significant production of secondaries. In the galaxy, the most dense non-collapsed objects are the molecular clouds, whose density can reach 10^4 protons/cm^3. The 2 objects form:



That is an ideal configuration, since

$$\text{CR collisions} \to \begin{cases} \pi^0 \to & \text{high energy } \gamma \\ \pi^\pm \to & \text{high energy } \nu_\mu, \nu_e \end{cases}$$

Thus, we expect to have γ *and* neutrinos.

There is some evidence that the system called RX J1713.7-3946 is a 'cosmic beam dump'. Indeed,
1) It is seen in X-rays, with many details
2) It is in Chinese Annales, 393 A.D. [13]
3) A molecular cloud seen in CO and 21 cm H line
4) And most interestingly, CANGAROO (since 2000, see [14]) and H.E.S.S. (since 2004, see [15]) do see TeV γ rays.
The distance is $L = 1$ kpc, the angular size about 1°, the density of the cloud ~ 100 part/cm^3. The source is transparent to gamma rays, so that the neutrino flux can be

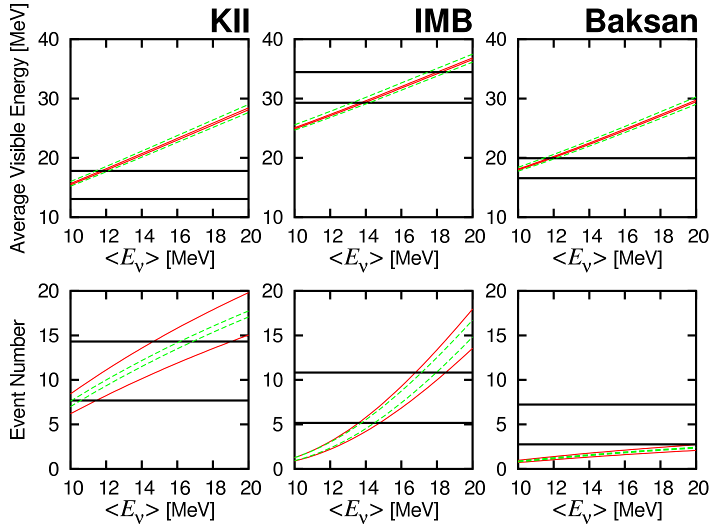


FIGURE 1. Average energies and number of events for three detectors. The horizontal lines describe the measured values, the oblique lines describe the theoretical expectations.

calculated easily and reliably as shown in the appendix. (However, no item above seems to be waterproof, what we discuss is just an appealing interpretation.)

We assume that the cosmic ray spectrum:

$$F_p = KE^{-\Gamma}, \quad \Gamma = 2 - 2.4$$

interacts with the molecular cloud and is the main source of visible gamma rays, through the reaction $p \to \pi^0 \to \gamma$. Neutrinos originate from similar processes, e.g., the reaction $p \to \pi^+ \to \mu^+ \to \nu_e$ will yield ν_e. Using the flux $F_\Gamma = 1.7 \cdot 10^{-11} (E/\mathrm{TeV})^{-2.2}$ $\mathrm{TeV}^{-1}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ measured by H.E.S.S. in the range between 1-40 TeV we get:

$$F^0_{\nu_\mu} = 7.3 \times 10^{-12} \left(\frac{E}{\mathrm{TeV}}\right)^{-2.2} \frac{1}{\mathrm{TeVcm^2s}}$$
$$F^0_{\overline{\nu}_\mu} = 7.4 \times 10^{-12} \left(\frac{E}{\mathrm{TeV}}\right)^{-2.2} \frac{1}{\mathrm{TeVcm^2s}}$$
$$F^0_{\nu_e} = 4.7 \times 10^{-12} \left(\frac{E}{\mathrm{TeV}}\right)^{-2.2} \frac{1}{\mathrm{TeVcm^2s}}$$
$$F^0_{\overline{\nu}_e} = 3.0 \times 10^{-12} \left(\frac{E}{\mathrm{TeV}}\right)^{-2.2} \frac{1}{\mathrm{TeVcm^2s}}$$

3 flavor oscillations take the simplest form (vacuum averaged or Gribov-Pontecorvo) and can be included easily. This leads to an important modification of the fluxes.

Finally we pass to the signals of neutrinos. ν interactions are due to deep elastic scattering. The simplest and most traditional observable is induced muons, that can be correlated to the source by mean of an angular cut. It is important to recall that high energy ν_μ are to some extent absorbed from the Earth, and that when the source is above the horizon, it is impossible to see anything due to the background from atmospheric μ. Along with oscillations, these effect decrease the observable neutrino signal. For an ideal detector, located in the Mediterranean, with Area=1 km^2, Data taking=1 year, $E_{thr.} = 50$ GeV we find that the number of expected events is about 10 (this was 30 if oscillations, absorption and μ-background were ignored, or even 40 if the slope was $\Gamma = 2.0$).

4. DISCUSSION AND PERSPECTIVES

We do not have a clear understanding of how SN explode. Perhaps, because it is a very difficult problem, or there is some missing ingredient, perhaps the answer is not unique (a combination of various mechanisms?), ... the confusion could persist even after next galactic SN.

νs from next galactic SN have an impressive potential to orient our understanding. The hypothesis of an "accretion phase", implying 10-20 % of the emitted energy, can certainly be tested. SN1987A does not contradict the 'delayed scenario' seriously (but does not help much either). There are chances to learn on oscillations and in general on particle physics. The possibility to use ν_e and neutral current events deserves consideration.

νs from SNR are an uncharted territory. Recent results from H.E.S.S. motivated us to consider one specific SNR (but we expect new results and surprises with γ rays). Sure enough, CR acceleration in SNR is not fully understood, and there are other possible sources for TeV ν astronomy, however the number of events we found (about 10/km^2 y) suggests the need of large exposures. It is important to improve our theoretical tools to describe SNR γ and ν.

APPENDIX: Cosmic rays and neutrinos. We report here on the details of the calculation of (anti) neutrino spectra from the decays of secondary particles produced

in pp interaction in astrophysical beam-dumps (table 1 of [4]). As a first approximation, we add the contributions to ν-emission from two chains: (1) $p \to m^{\pm} \to (anti)\nu$ and (2) $p \to m^{\pm} \to \mu^{\pm} \to (anti)\nu$, where $m^{\pm} = \{\pi^{\pm}, K^{\pm}\}$, $\nu = \{\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu\}$.

If we assume scaling, and take a power-law proton spectrum $F_p(E_p) = KE^{-\Gamma}$, we can express the (anti) neutrino spectra from (1) and (2) as:

$$F_\nu^{(1),(2)}(E_\nu) = \frac{\Delta X}{\lambda_p} \cdot \psi_\nu^{(1),(2)} \cdot F_p(E_\nu) \tag{4}$$

where ψ_ν indicate the neutrino emissivity coefficient, defined in [16], ΔX the column density traversed by the protons and λ_p the interaction length of CR. Similarly, the γ-spectrum from $p \to \pi^0 \to \gamma$ is:

$$F_\gamma(E_\gamma) = \frac{\Delta X}{\lambda_p} \cdot \psi_\gamma \cdot F_p(E_\gamma), \text{ where } \psi_\gamma = \frac{2Z_{p\pi_0}}{\Gamma} \tag{5}$$

In equations 4 and 5 we assume a thin target. When we assume scaling, following Gaisser (see [17], formulas 4.1 and 4.2 in approximation 3.31) we have:

$$F_{\nu_\mu}^{(1)} = \Delta X \sum_{m^+=\pi^+,K^+} \int_{\frac{E_{\nu_\mu}}{1-r_m}}^{\infty} \frac{dn_{\nu_\mu m^+}}{dE_{\nu_\mu}} D_{m^+}(E_{m^+})\, dE_{m^+} \tag{6}$$

with:

$$\frac{dn_{\nu_\mu m^+}}{dE_{\nu_\mu}} = \frac{B_{m^+}}{1-r_m} \frac{1}{E_{m^+}},\ D_{m^+} = \frac{Z_{pm^+}}{\lambda_p} F_p(E_{m^+}) \tag{7}$$

where B_{m^+} is the branching ratio for $m^+ \to \mu^+\nu_\mu$, $r_m = m_\mu^2/m_m^2$ (subscript $m = \{\pi, K\}$), and Z_{pm^+} are the spectrum-weighted momenta, defined in [17]. We recall that in the scaling hypothesis the spectrum-weighted momenta depend only from the spectral index Γ. We can get the $\bar{\nu}_\mu$ spectrum from the same formula, replacing m^+ with m^-. Evaluating the integrals, we get:

$$F_{\nu_\mu}^{(1)} = \frac{\Delta X}{\lambda_p} F_p(E_{\nu_\mu}) \sum_{m^+=\pi^+,K^+} B_{m^+} \frac{Z_{pm^+}}{\Gamma} (1-r_m)^{\Gamma-1} \tag{8}$$

and so:

$$\psi_{\nu_\mu}^{(1)} = \sum_{m^+=\pi^+,K^+} B_{m^+} \frac{Z_{pm^+}}{\Gamma} (1-r_m)^{\Gamma-1} \tag{9}$$

Similarly, for $\psi_{\bar{\nu}_\mu}^{(2)}$ we have (see [17], formula (7.14), in which we replace ϕ_π with the factor $[\Delta X Z_{pm^-} B_{m^-} F_p(E_{\nu_\mu})]/\lambda_p$, and then divide by $\Delta X F_p/\lambda_p$):

$$\psi_{\bar{\nu}_\mu}^{(2)} = \sum_{m^-=\pi^-,K^-} Z_{pm^-} B_{m^-} \frac{1-r_m^\Gamma}{\Gamma(1-r_m)} \left\{ \langle y_{0\nu_\mu}^{\Gamma-1} \rangle + \right.$$
$$\left. + \left[1 + r_m - \left(\frac{2\Gamma r_m}{1-r_m^\Gamma} \right) \frac{1-r_m^{\Gamma-1}}{\Gamma-1} \right] \frac{\langle y_{1\nu_\mu}^{\Gamma-1} \rangle}{1-r_m} \right\} \tag{10}$$

where the moments:

$$\langle y_{i\nu}^{\Gamma-1} \rangle = \int_0^1 y^{\Gamma-1} g_{i\nu}(y) dy \tag{11}$$

for (anti)ν_e and (anti)ν_μ are given in table 7.3 from [17] (the $g_{i\nu}(y)$ are given in table 7.2 from [17]). We can obtain $\psi_{\bar{\nu}_\mu}^{(2)}$ from eq. 10, replacing Z_{pm^-} with Z_{pm^+}. In our approximation, the (anti)ν_e emissivity coefficients are given by eq. 10, inserting respectively Z_{pm^+} for ψ_{ν_e}, and Z_{pm^-} for $\psi_{\bar{\nu}_e}$ and the appropriated moments. The numerical values of $k_\nu = (\psi_\nu^{(1)} + \psi_\nu^{(2)})/\psi_\gamma$ for several values of Γ, obtained using the spectrum-weighted momenta from figure 5.5 by [17], are reported in table 1 by [4].[3]

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[3] In our calculation, we omitted the contribution of ν's produced in other decay modes of π and K, as well as in K^0's decay chains, and so we introduce and error at several % level. In the calculation made in [16], those contributions are taken into account. Moreover, the values of Z_{pm} used in [16] differ from our values at several % level. We note that in the calculation made in [16], only the first addend in the parenthesis, $\langle y_0^{\Gamma-1} \rangle$, is taken into account, that introduces an error at the few % level.